

Exercise 14

For the following exercises, find the average rate of change of each function on the interval specified for real numbers b or h .

$$r(t) = 4t^3 \text{ on } [2, 2 + h]$$

Solution

The average rate of change of the function on $[2, 2 + h]$ is

$$\begin{aligned} \frac{r(2 + h) - r(2)}{(2 + h) - 2} &= \frac{4(2 + h)^3 - 4(2)^3}{h} \\ &= \frac{4(2^3 + 2^2 \cdot 3h + 2 \cdot 3h^2 + h^3) - 4(8)}{h} \\ &= \frac{4(8 + 12h + 6h^2 + h^3) - 4(8)}{h} \\ &= \frac{32 + 48h + 24h^2 + 4h^3 - 32}{h} \\ &= \frac{48h + 24h^2 + 4h^3}{h} \\ &= 48 + 24h + 4h^2. \end{aligned}$$